Variation of mass with velocity:

There is variation of mass with velocity in relativity, that is mass varies with the velocity, when the velocity is comparable with the velocity of the light.

Let, there are two inertial frames of references S and S'. S is the stationary frame of reference and S' is the moving frame of reference. At time $t = t' = 0$ that is in the start, they are at the same position that is observers O and O' coincides.

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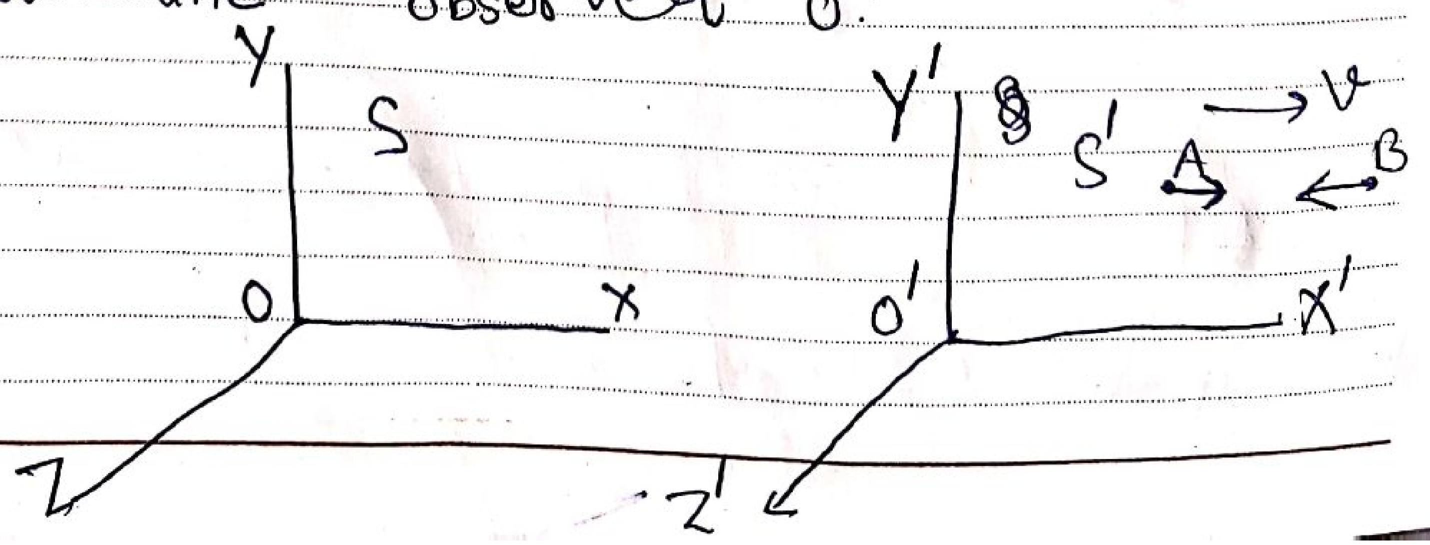
After that S' frame starts moving with a uniform velocity v along x-axis.

Suppose, there are two particles moving in opposite direction in frame S'. velocity of particle A will be u' and of B will be $-u'$ according to the observer O'.



Let us study the velocities and mass of these particles from frame S.
velocity of A is u_1 and B is u_2 from frame S and these are given by relativistic addition of velocity relation respectively.

$$u_1 = \frac{(u' + v)}{(1 + u'v/c^2)} \quad \text{——— (1)}$$

$$u_2 = \frac{(-u' + v)}{\left(1 - \frac{u'v}{c^2}\right)} \quad \text{——— (2)}$$

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Let, m_1 and m_2 are the mass of A and B from frame S respectively.
As the particles are moving to each other, at certain instant they will Collide and Came to rest. But even when they came to rest, they travel with the velocity of the frame S' that is with v.

According to the law of conservation of momentum:

Momentum before Collision = momentum after Collision

Thus,

$$m_1 u_1 + m_2 u_2 = (m_1+m_2)v = m_1 v + m_2 v$$

or, $$m_1(u_1 - v) = m_2(v - u_2) \quad \text{--- (3)}$$

put eqs. (1) and (2) in eq^n (3), we get

$$m_1\left[\frac{(u'+v)}{(1+u'v/c^2)} - v\right] = m_2\left[v - \frac{(-u'+v)}{\left(1-\frac{u'v}{c^2}\right)}\right]$$

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$$m_1\left[\frac{1}{\left(1+\frac{u'v}{c^2}\right)}\right] = m_2\left[\frac{1}{\left(1-\frac{u'.v}{c^2}\right)}\right]$$

$$\frac{m_1}{m_2} = \frac{\left(1+\frac{u'v}{c^2}\right)}{\left(1-\frac{u'v}{c^2}\right)} \quad \text{--- (4)}$$

Now, square eq^n (1), then divide both sides by c^2 and subtract both sides by 1, we get

$$1 - \frac{u_1^2}{c^2} = 1 - \left[(u'+v)/c/(1+u'v/c^2)\right]^2$$

$$1-\frac{u_1^2}{c^2} = \frac{\left(1+ u'^2v^2/c^4 - u'^2/c^2 - \frac{v^2}{c^2}\right)}{(1+u'v/c^2)^2} \quad \text{--- (5)}$$

Similarly, by squaring eq^n (2), then dividing both sides by c^2 and subtracting both sides by 1, we get

$$1-\frac{u_2^2}{c^2} = \frac{\left(1+ u'^2v^2/c^4 - u'^2/c^2 - v^2/c^2\right)}{\left(1-\frac{u'.v}{c^2}\right)^2} \quad \text{--- (6)}$$

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on dividing eq^n (6) by (5), we get

$$\frac{\left(1-\frac{u_2^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)} = \frac{\left(1+\frac{u'v}{c^2}\right)^2}{\left(1-\frac{u'v}{c^2}\right)^2}$$

Take square root on both sides,

$$\frac{\left(1-\frac{u_2^2}{c^2}\right)^{1/2}}{\left(1-\frac{u_1^2}{c^2}\right)^{\frac{1}{2}}} = \frac{\left(1+\frac{u'v}{c^2}\right)}{\left(1-\frac{u'v}{c^2}\right)} \quad \text{--- (7)}$$

now, Compare eq^n 4 and 7, we get

$$\frac{m_1}{m_2} = \frac{\left(1 - \frac{u_2^2}{c^2}\right)^{1/2}}{\left(1 - \frac{u_1^2}{c^2}\right)^{1/2}} \quad \text{--- (8)}$$

To make this result simple, let us assume that the particle B is in the state of rest from frame S i.e. it has zero velocity before Collision,

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Thus, $u_2 = 0$

and, $m_2 = m_0$

Where m_0 is the rest mass of the particle,

Therefore, eq^n 8 becomes,

$$\frac{m_1}{m_0} = \frac{1}{\sqrt{1 - \frac{u_1^2}{c^2}}}$$

also, assume $u_1 = v$ and $m_1 = m$

Then,

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or, $$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}} \quad \text{--- (9)}$$

This eq^n (9) represents the eq^n of the variation of mass with the velocity.